

6 Power Set

44. Let $S = \{1, 2, 3\}$. Which of the following statements are true (if any exist)? For each statement, explain why it is true or false.

- (a) $S \subseteq \mathcal{P}(S)$.
- (b) $\mathcal{P}(S) \subseteq S$.
- (c) $S \in \mathcal{P}(S)$.
- (d) $(\forall x)(x \in S \Rightarrow x \in \mathcal{P}(S))$.
- (e) $(\exists x)(x \in S \Rightarrow x \in \mathcal{P}(S))$.

45. Justify whether the following statement holds for any sets A and B :

$$A \subseteq B \Rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B).$$

46. Does $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$ hold? Provide a detailed justification.

47. Justify whether the following statement is true or false:

If $\mathcal{P}(X) = \mathcal{P}(Y)$ for sets X and Y , then X and Y are equal.

48. Let A be a non-empty set. Determine which of the sets

$$\emptyset, \{\emptyset\}, A, \{A\}, \{A, \emptyset\}$$

are elements of and which are subsets of the sets $\mathcal{P}(A)$ and $\mathcal{P}(\mathcal{P}(A))$.

7 Cartesian Product of Sets

49. Does $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$ hold? Provide a detailed justification.

50. Justify whether the following statement holds for any sets A , B , and C :

$$A \times (B \cup C) = (A \times B) \cup (A \times C). \tag{2}$$

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.